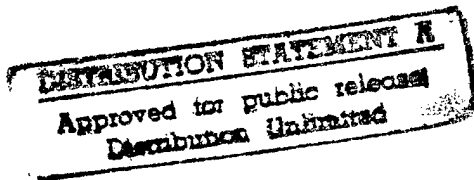0031

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THE EFFECT OF NONLINEARITIES ON FLEXIBLE COMPOSITE STRUCTURES

by

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The present report attempts to present a comprehensive review of achievements made under the support of AFOSR on the effect of nonlinearities on flexible composite structures. The research was focused on the mechanisms by which energy is transferred from high- to low-frequency modes. The report is focused on some recent experiments, conducted with the AFOSR support, that reveal a perspective of the mechanisms by which energy is transferred from high- to low-frequency modes is presented. The focus is on some recent experiments that reveal how a low-amplitude high-frequency excitation can produce a large-amplitude low-frequency response. Such a phenomenon is potentially harmful, if not catastrophic. Specifically, these experiments clarify the role of internal resonances, combination external and parametric resonances, and the interactions among modes of widely spaced frequencies.

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1 Background

An understanding of the dynamic characteristics of a structural system is essential for its design and control. Many of the important characteristics can only be modeled by nonlinear governing equations. When the governing equations are nonlinear, the system and the characteristics of the solutions are also said to be nonlinear. There is vast range of interesting, important, and potentially dangerous phenomena that are nonlinear. Nonlinearities can have important influences even while the amplitudes of the response are quite small (Nayfeh and Mook, 1979). Here a limited perspective on some recent developments is presented.

As an example, modeling a system that is subjected to a parametric excitation by linear equations and boundary conditions is unrealistic if the parametric excitation leads to instability because such a model predicts that the growth of the response is exponential. Consequently, it would be more realistic to include nonlinear terms, which limit the predicted response. Moreover, the linear model may predict stability (i.e., a decaying response), but the actual response may not decay under certain conditions. In this case, the parametric excitation produces a so-called subcritical instability that is only predictable by a nonlinear model.

Nonlinearity brings a whole range of phenomena that are not found in linear systems. In single-degree-of-freedom systems, these include multiple solutions and jumps; limit cycles; frequency entrainment; natural-frequency shifts; subharmonic, superharmonic, combination, and ultrasubharmonic resonances; period-multiplying and demultiplying bifurcations; and chaos. The devastating consequences of having one harmonic load with a frequency near the natural frequency might be lowered to a tolerable level by simply adding one or more nonresonant harmonic loads that produce a shift in the natural frequency. The large response produced by a primary resonant excitation can also be significantly reduced by simply adding other superharmonic-resonant loads having the proper amplitudes and phases. When the nonlinear terms are cubic and the sum of three frequencies in a multifrequency load equals or nearly equals the natural frequency, the system can experience a combination-resonant response in which the peak amplitudes are several times larger and appear more often than those predicted by linear theory. For such a response, the actual fatigue life can be much lower than what was predicted.

A successful strategy for solving the nonlinear partial-differential equations governing beams, plates, shells, etc. is to discretize the equations by writing the deflections as expansions in terms of the linear, free-vibration modes. The time-dependent coefficients in these expansions (the generalized coordinates) are governed by a system of nonlinear, coupled, second-order, ordinary-differential equations. Typically, the coupling is through the nonlinear terms, but in some cases there are repeated natural frequencies and the Jordan form for the linear terms has coupling (i.e., off-diagonal) terms. For these as well as other multi-degree-of-freedom systems, another example of a nonlinear phenomenon is an interaction among different modes, which can result in an energy exchange among the modes. What makes the exchange of energy among modes dangerous is that typically energy is transferred from the low-amplitude high-frequency components of the motion associated with the high modes to the high-amplitude low-frequency components of the motion associated with the low modes. Thus, the modal interaction makes it possible for a high-frequency low-amplitude excitation, which is capable of doing a lot of work on the structure in a short period, to produce a large-amplitude low-frequency response. In the absence of modal interactions, the steady-state response of a damped structure will consist of only the directly excited modes.

Nonlinear modal interactions have been the subject of a great deal of recent research. It has been found that, in weakly nonlinear systems where there exists a special relationship between two or more natural frequencies and an excitation frequency, the long-time response can contain large contributions from many linear modes (Nayfeh and Mook, 1979; Nayfeh and Balachandran, 1989).

The large presence of more than one mode generally makes the actual response more complicated. increases the number of modal equations that must be treated. and causes the analysis to be more difficult. Modal interactions can lead to dangerously large responses in the very modes that are insignificant according to linear analysis.

Most of the research on modal interactions is focused on autoparametric. also called internal. resonances in systems where the linear natural frequencies are commensurate or nearly commensurate. Autoparametric resonances have been successfully treated with perturbation methods. There also exists a large body of experimental results which are in good general agreement with the perturbation results. Autoparametric resonances may provide coupling and energy exchanges among the modes. Consequently, exciting a high-frequency mode may produce a large-amplitude response in a low-frequency mode involved with it in an autoparametric resonance.

In externally excited multi-degree-of-freedom systems. combination resonances may occur in response to a simple-harmonic excitation. Consequently. a high-frequency excitation may produce large-amplitude responses in the low-frequency modes that participate in the combination resonance. In parametrically excited systems. multimode interactions can occur when the excitation frequency is near the sum or difference of two or more linear natural frequencies. These so-called combination resonances have been studied extensively in the literature (Evan-Iwanowski. 1976; Nayfeh and Mook, 1979). Again. these combination resonances can lead to interactions between high- and low-frequency modes. Such transfers of energy are predictable with the current state of analytic methods.

Often. when the response of a system excited at a high frequency becomes chaotic. low-frequency modes are excited. Haddow and Hasan (1988) parametrically excited a cantilever beam near twice the natural frequency of its fourth mode. While increasing the excitation frequency. they observed that a planar periodic response consisting essentially of the fourth mode lost stability. and a nonplanar chaotic motion developed. and that the energy in the beam seemed to cascade down through the modes. resulting eventually in a very low-frequency high-amplitude steady-state response. Burton and Kolowith (1988) conducted an experiment similar to that of Haddow and Hasan. In certain regions of the parameter space. they observed chaotic motions where the first seven in-plane bending modes as well as the first torsional mode were present in the response. Cusumano and Moon (1990) conducted an experiment with an externally excited cantilever beam. They observed a cascading of energy into low-frequency components in the response associated with chaotic nonplanar motions.

Several recent experimental studies suggest that another type of interaction may occur between high- and low-frequency modes. In these experiments. a high-frequency mode was directly excited either parametrically or externally. yet the response contained a large contribution from the first mode. The presence of the first mode was accompanied by a slow modulation of the amplitude and phase of the high-frequency mode with the frequency of the modulation being nearly equal to the natural frequency associated with the first mode. The results indicate that the mechanism for the excitation of the first mode is neither a classical internal resonance nor an external or a parametric combination resonance involving the first mode. Rather. the appearance of the first mode is accompanied by a slow modulation of the high-frequency modes.

The interaction between high- and low-frequency modes observed experimentally is of great practical importance. In many manufacturing structures and dwellings. high-frequency excitations can be caused by rotating machinery; in large floating structures. high-frequency excitations can be caused by waves; in ships. high-frequency excitations can be caused by the propeller blades passing the rudder; etc. Through the mechanisms discussed above. energy from high-frequency sources can be transferred to the low-frequency modes of supporting structures or foundations. and the result can be harmful large-amplitude oscillations. Moreover. some preliminary results (Nayfeh

and Nayfeh, 1993) indicate that the use of conventional methods for decreasing modal interactions, such as increasing the dissipation or decreasing the forcing amplitude, may have undesirable effects.

In the present paper a limited discussion of the mechanisms for the transfer of energy from high- to low-frequency modes is presented. In Section 2 internal resonances are considered, in Section 3 combination resonances are considered, and in Section 4 interactions among widely spaced modes are considered. Due to the page limitation, the discussion is brief; it is focused on some very recent work conducted at VPI&SU.

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2 Other Developments Resulting from the AFOSR Sponsored Research

2.1 Internal Resonances

Internal resonances are responsible for many interesting, unusual, and dangerous phenomena. For example, they are responsible for the instability of the planar motions of a string or a symmetric beam excited by a harmonic planar force. Experiments show, and nonlinear analyses predict, that the response of a string or a symmetric beam to a plane harmonic excitation is planar provided the excitation amplitude is below a critical value. Above the critical value, the planar motion becomes unstable and gives way to a nonplanar, whirling motion. The whirling motion is a direct consequence of the fact that the natural frequency for motion in the plane of the excitation is the same as the natural frequency for motion in the plane perpendicular to the plane of the excitation.

When two natural frequencies of a system with quadratic nonlinearities are in the ratio 2:1, there exists a saturation phenomenon. When the system is excited at a frequency near the higher natural frequency, the structure responds at the frequency of the excitation and the amplitude of the response increases linearly with the amplitude of the excitation, in perfect agreement with

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the predictions of linear theory. Then, when the amplitude reaches a critical value, this response loses stability, the lower mode is strongly excited, and the ensuing motion is dominated by a very large contribution from this mode at half the frequency of the excitation. On some occasions in the laboratory, the structural model was destroyed by the large-amplitude motions. On other occasions it was observed that the amplitude of the higher mode did not increase with further increases in the amplitude of the excitation: the higher mode became saturated and all additional energy added to the structure by the excitation overflowed into the lower mode.

An interesting phenomenon that could be related to internal resonance was reported by Cole (1990). While investigating the influence of spoiler surfaces on the aeroelastic behavior of low-aspect-ratio rectangular wings, Cole encountered what he called a "torsional instability". He found that above a certain spoiler size, the wing experienced torsional instability at dynamic pressures well below the expected flutter conditions.

"Torsional instability" is represented in Fig. 1, which was taken from Cole's paper. In the figure "h_s" refers to the height of the spoiler, which runs in the spanwise direction at 0.1 chord from the leading edge, and w refers to its spanwise dimension, see Fig. 2. In part a, one sees that, for $h_s = 1$, a sudden and dramatic loss of stability occurs when the Mach number M is near 0.7. Flutter occurs at a dynamic pressure very far below what was expected. The motion that develops "appears to be nearly single-degree-of-freedom torsional instability...". In part b, one can see that the frequency of the flutter motion suddenly increases near $M = 0.7$ and is near the free-vibration (i.e., wind-off) torsional frequency. The lowest five experimentally obtained natural frequencies are 2.85 Hz (first bending), 13.1 Hz (first torsion), 15.5 Hz (in-plane), 26.2 Hz (second bending), and 62.6 Hz (second torsion). Hence, instead of flutter, the wing suddenly experiences a strong response to the buffeting from the wake of the spoiler. At lower Mach numbers Cole reported that the motion appears to be the classical coupling of the first bending and first torsional modes. Indeed, at $M = 0.6$ the observed frequency is about 7 Hz or 8 Hz, between the first bending and first torsional frequencies. We note that the frequency of the first torsional mode is exactly one-half the frequency of the second bending mode. Hence, there is an internal resonance, which to the accuracy reported is perfectly tuned, and hence it is possible that modal interactions between these modes were responsible for the observed torsional instability.

To investigate whether the two-to-one frequency ratio may be responsible for the torsional instability observed in the wind-tunnel, Oh, Nayfeh, and Mook (1994) fabricated an aluminum plate having the same dimensions as those of the plate tested by Cole. The plate was attached in a cantilever mode to a 2000-lb shaker and excited harmonically at the base near the frequency of the second bending mode. During sweeps of the excitation frequency or amplitude upward the first torsional mode of the plate was excited. This phenomenon was identified by time histories, power spectra, and state-space plots and was further characterized by force-response plots. In Fig. 3, typical power spectra are shown for the excitation and plate response when the torsional mode was excited. These results illustrate a possible mechanism for the torsional instability observed by Cole (1990) in his wind-tunnel experiments.

In the laboratory, the second bending mode was excited by a shaker. In Cole's experiment, the second bending mode was most likely excited by pressure fluctuations in the turbulent wake of the spoiler. In both cases, it seems that energy was transferred from a high-frequency mode to a low-frequency mode by means of an internal resonance. Such a transfer is potentially dangerous because, for a given amount of energy, a reduction in frequency must be accompanied by an increase in amplitude.
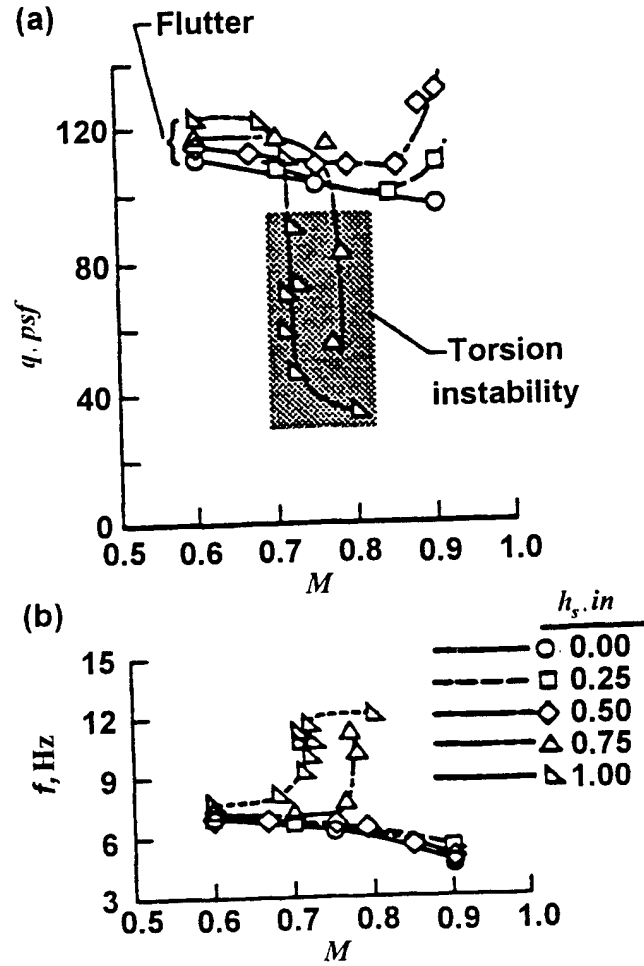
Figure 1: Influence of spoilers on the flutter of a model of a wing: (a) variation of the flutter dynamic pressure with Mach number and (b) variation of the flutter frequency with Mach number (Cole, 1990).
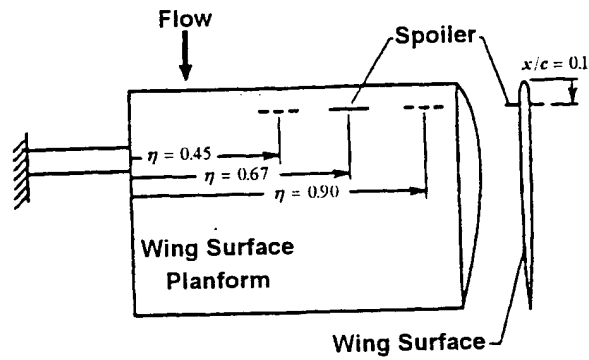

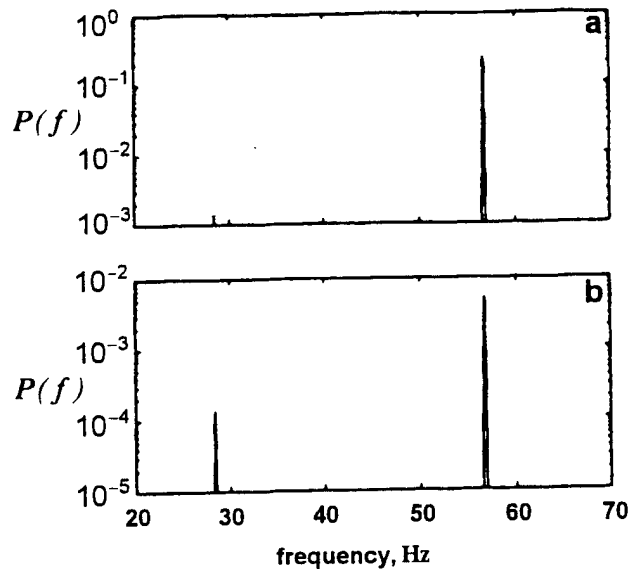Figure 2: A model of a wing with spoilers (Cole, 1990).

Figure 3: Activation of a two-to-one internal resonance in a cantilever metallic plate: (a) power spectrum of the input and (b) power spectrum of the response (Oh et al., 1994).

2.2 Combination Resonances

Dugundji and Mukhopadhyay (1973) conducted experiments on a cantilever beam subjected to a parametric base excitation at a frequency close to the sum of the natural frequencies of the first bending and first torsional modes, which are approximately in the ratio of 18 to one. They found that the high-frequency excitation can produce a large-amplitude response in a low-frequency (in this case, the first bending) mode. Three recent experiments indicate that activation of a low-frequency mode by a high-frequency excitation is generic to a very broad class of dynamic systems.

2.2.1 A Parametrically Excited Cantilever Beam

Anderson, Balachandran, and Nayfeh (1994) parametrically excited a metallic cantilever beam. The lowest four natural frequencies of the beam are 0.65, 5.60, 16.19, and 31.91 Hz. The beam was slightly bent. The excitation frequency was chosen to be the control parameter, while the base acceleration was held constant at 2.0 g rms, where g stands for the acceleration due to gravity and rms denotes the root mean square. Initially, the excitation frequency was set at 32.50 Hz. The response consisted of contributions from the fourth and third modes only. They were excited by a primary resonance and a principal parametric resonance, respectively. As the excitation frequency was decreased to 32.0 Hz, the power spectrum of the response (Fig. 4a) consisted of three peaks: a peak at the excitation frequency, a peak corresponding to the fourth mode, and a peak corresponding to the first mode. In Fig. 4b we show a two-dimensional projection of a Poincaré section of the attractor. It consists of a closed loop of points, indicating that the motion is two-period quasiperiodic.

In this experiment, the beam was excited at a high frequency, yet the response contained a large-amplitude first-mode contribution. The activation of the first mode is the result of a combination resonance of the additive type.
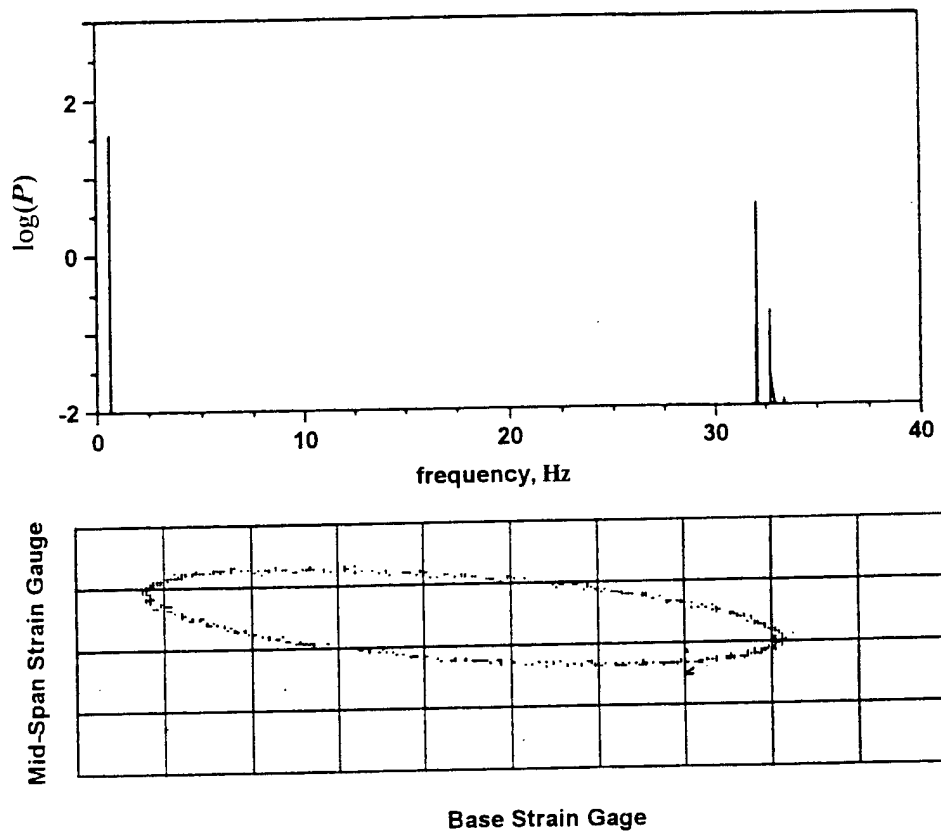
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Figure 4: Combination resonance of the additive type in the response of a cantilever metallic beam: (a) power spectrum of response and (b) Poincaré map of response (Anderson et al. 1994).
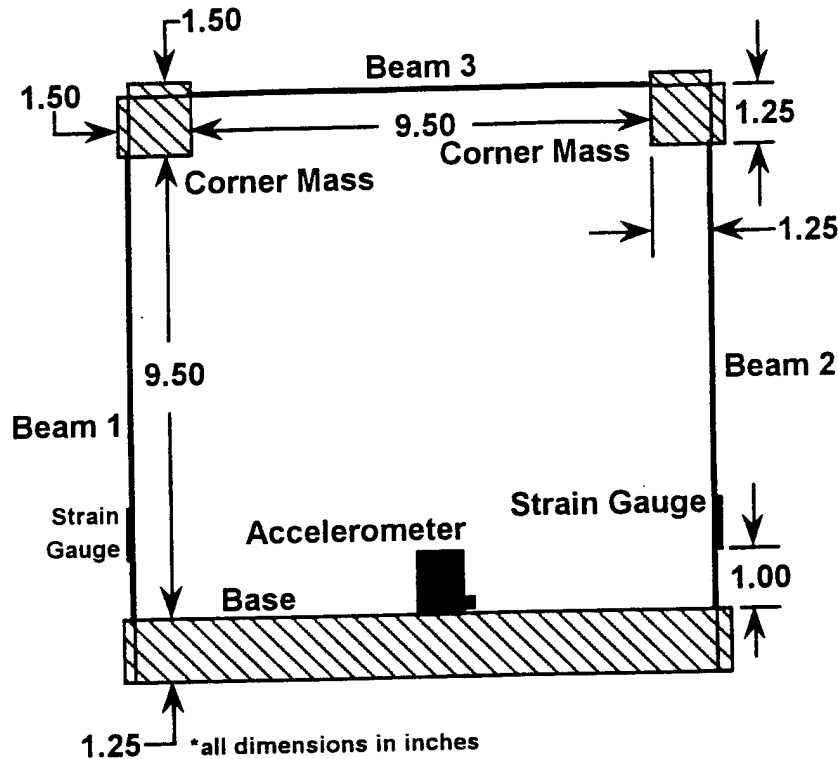
Figure 5: A three-beam two-mass structure (Popovic et al., 1994).

2.2.2 Frames

In the second experiment. Popovic, Nayfeh, Oh. and Nayfeh (1994) tested the three-beam-frame structure shown in Fig. 5. It consisted of three steel beams connected by a base and two aluminum corner masses of 8.98×10^{-3} slug. The width and thickness of the steel beams are 0.75" and 0.035". To monitor the response of the structure, they located a strain gage on the outer side of each of the two vertical beams. approximately one inch from the bottom. They used an accelerometer attached to the base of the structure to monitor the excitation. The resolution of the accelerometer was ± 0.005 g. The experimental apparatus included a 250-lb shaker and the data acquisition instrumentation. The base of the structure was bolted to the shaker top. Experimental modal analysis was used to determine the linear natural frequencies of the frame. The lowest eight identified frequencies are 3.7, 23.8, 43.0, 68.7, 82.8, 101.8, 190.8, and 195.6 Hz. It is clear that this frame has widely spaced frequencies and some of the frequencies are nearly commensurate.

Setting the excitation at 1.2 g and 194.2 Hz. Popovic et al. observed significant responses at 3.7, 190.5, and 194.2 Hz. The spectrum of the response is shown in Fig. 6. The peaks at 3.7 Hz and 190.5 Hz represent contributions from the first and seventh mode. The driving frequency is the sum of the frequencies of the two peaks at 3.7 Hz and 190.5 Hz. Thus, the first mode was activated by a combination resonance of the additive type. Again. in this experiment, exciting the frame with a high-frequency excitation resulted in a large-amplitude first-mode response.

2.2.3 Composite Plates

Oh and Nayfeh (1995b) tested a six-layer symmetric laminated composite cantilever plate at its base. The plate has the layup $(90/30/-30/-30/30/90)_s$. The lowest seven experimentally obtained
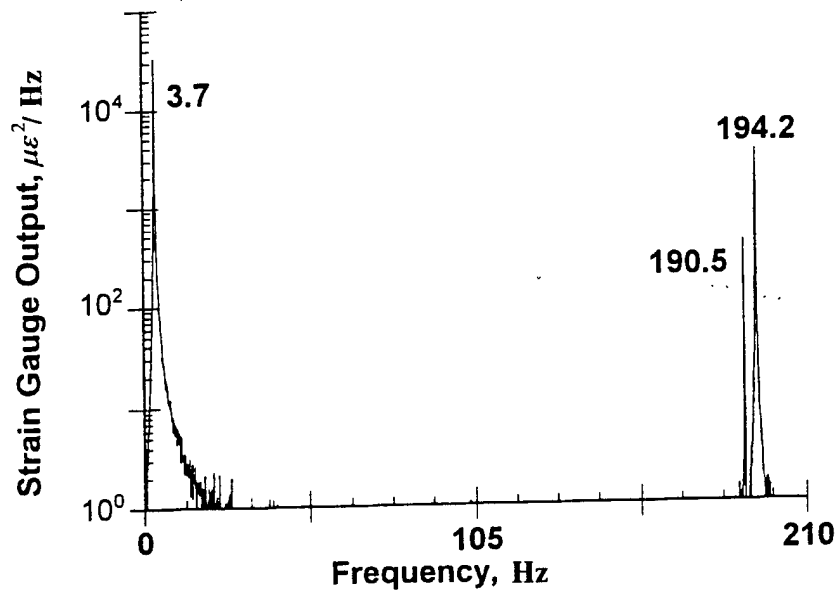
Figure 6: Power spectrum of the response of the frame. showing the activation of a combination resonance of the additive type (Popovic et al.. 1994).

natural frequencies are 42.79, 145.75, 275.11. 489.68. 734.66. 798.03. and 1001.4 Hz. The input was monitored using an accelerometer mounted on the base. and the response was monitored using a laser vibrometer.

In one test. Oh and Nayfeh (1995b) set the excitation at 13.66 g rms and 1133.79 Hz: the latter is approximately the sum of the frequencies of the first and seventh modes. In Fig. 7. characteristics of the plate response are shown. The time trace and power spectrum of the response in Figs. 7a and b show that the plate motion is dominated by the first mode. It is clear from Fig. 7b that the contributions of the first and seventh modes to the plate response are much larger than the contribution at the excitation frequency. The power spectrum indicates that the plate motion has contributions from two independent frequencies. and hence the motion may be periodic with a large period or quasiperiodic. A pseudo-state plane was reconstructed by using the time delay dt. corresponding to the first zero crossing of the autocorrelation function of the response. The result shown in Fig. 7c suggests that the motion is quasiperiodic. The closed curve in the Poincaré section in Fig. 7d confirms the quasiperiodicity of the motion.

2.3 Energy Exchanges Among Widely Spaced Modes

In four recent experiments a new mechanism for the transfer of energy from high- to low-frequency modes has been observed. This mechanism is neither a classical internal resonance nor an external or parametric resonance involving the low-frequency mode. Rather. the appearance of the low-frequency mode is accompanied by a slow modulation of the amplitude and phase of the high-frequency mode.

2.3.1 Experiments on a Parametrically Excited Cantilever Beam

In the first experiment. Anderson. Balachandran. and Nayfeh (1992) parametrically excited a metallic cantilever beam. The beam was slightly bent in the static configuration. The first four experimentally identified natural frequencies of the beam are 0.65. 5.65. 16.19. and 31.91 Hz.
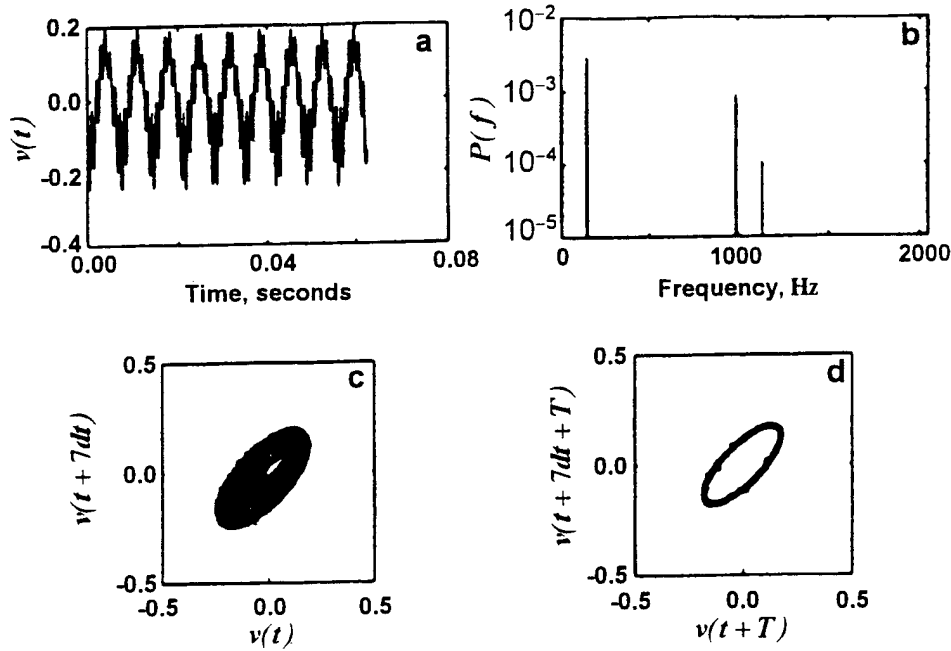
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Figure 7: Activation of a combination resonance of the additive type in the response of a quasisotropic composite plate: (a) time trace. (b) power spectrum. (c) pseudo-state portrait. and (d) Poincaré map (Oh and Nayfeh, 1995b) .

The base motion was monitored with an accelerometer. The response was monitored with two strain gages: one located at $x/L = 0.06$ and the other located at $x/L = 0.25$. where x is the distance along the undeformed beam measured from the base and L is the length of the beam.

The accelerometer and strain-gage spectra were monitored as the excitation frequency was varied. For the spectral analyses. there were 1280 lines of resolution in a 40 Hz baseband. A flat top window was used during periodic excitations. and a Hanning window with thirty overlap averages was used during random excitations. The two strain-gage signals were plotted against each other on the digital oscilloscope. thereby producing a pseudo-state plane. For the Poincaré sections (Moon, 1992: Nayfeh and Balachandran, 1994). one half of the excitation frequency was used as the clock frequency. Fourier spectra. pseudo-state planes. autocorrelation functions. and dimension calculations were used to analyze the different motions.

The excitation frequency was chosen to be the control parameter. and the base acceleration was held constant at 0.85 g rms, where g stands for the acceleration due to gravity and rms denotes root mean square. Initially when the excitation frequency f_e was 33.5 Hz. there was a peak in the response spectrum at the excitation frequency. This peak is due to a primary resonance of the fourth mode. As the excitation frequency was gradually decreased to 32.31 Hz. the third mode appeared in the response. It was excited by a principal parametric resonance. The response spectrum, Fig. 8. has peaks at f_e and $f_e/2$.

When the excitation frequency was decreased to 32.298 Hz. the periodic response consisting of the third and fourth modes lost stability, and a modulated motion developed in which the amplitudes and phases varied with time. The spectrum of this response is shown in Fig. 9, with the sidebands around the carrier frequency at $f_e/2$ indicating a modulated motion. The sideband spacing f_m is 0.58 Hz, which is close to the first natural frequency of the beam. Once the modulated motions developed. Anderson et al. (1992) observed that the contribution of the first mode to the
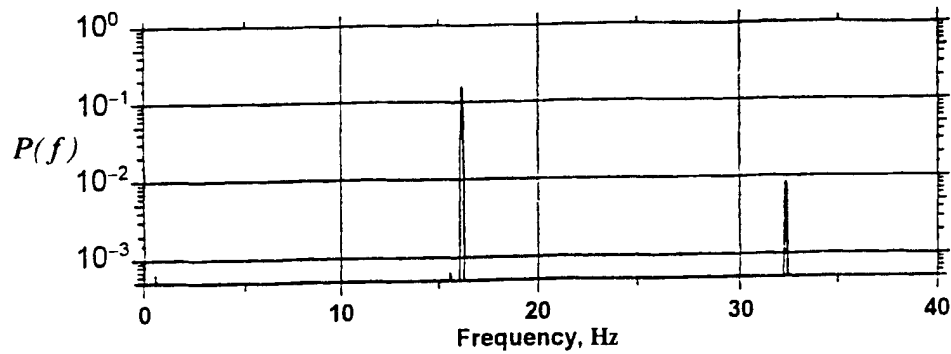
Figure 8: Power spectrum of the response of a metallic cantilever beam to a principal parametric excitation of the third mode (Anderson et al., 1992).
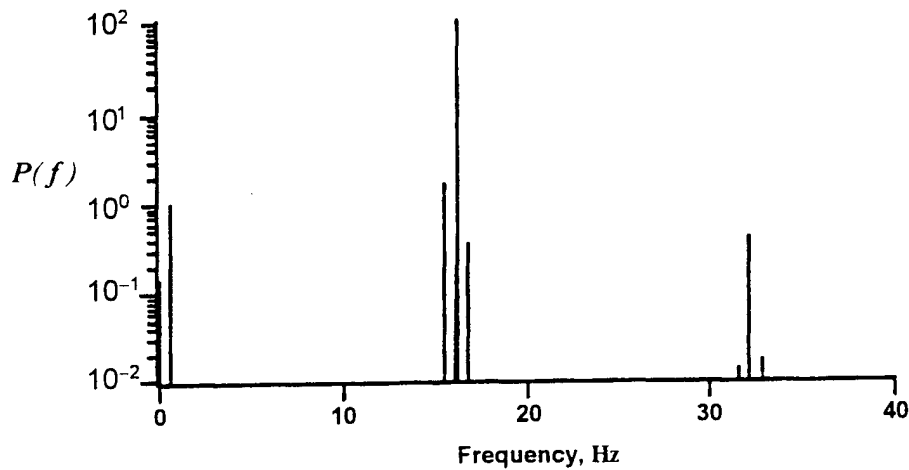

Figure 9: Power spectrum of the modulated response showing a transfer of energy from a low-amplitude high-frequency excitation to a high-amplitude low-frequency mode in the response of a parametrically excited cantilever beam (Anderson et al., 1992).

response became large. During the experiments, the presence of the first mode was very apparent visually.

When the excitation frequency was further reduced to 32.289 Hz, the motion appeared to become chaotic with a large out-of-plane component. The spectrum, Fig. 10, has a continuous character in many frequency bandwidths, which is a characteristic of chaotic motion (Moon, 1992; Nayfeh and Balachandran, 1994).

In Fig. 11, a time record of the strain obtained during the transient phase of the motion when f_e was changed from 32.298 Hz to 32.289 Hz is shown. During the initial phase, the third and fourth modes dominate the response. Subsequently, there is a transition from the response composed mainly of modulated high-frequency (fast-time scale) motion to one dominated by the low-frequency (slow-time scale) first mode. The displacements observed during the low-frequency dominated phase of the motion are much larger than those observed during the high-frequency dominated phase. The change in the time history is striking.

In summary, in these experiments, a beam was subjected to a simple-harmonic, low-amplitude excitation. The frequency of the excitation was near the fourth natural frequency;
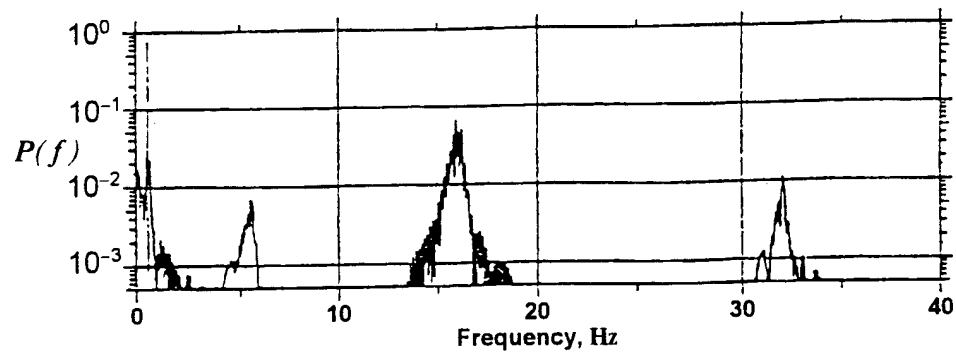
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Figure 10: Power spectrum of a chaotically modulated response of a parametrically excited cantilever beam (Anderson et al., 1992).
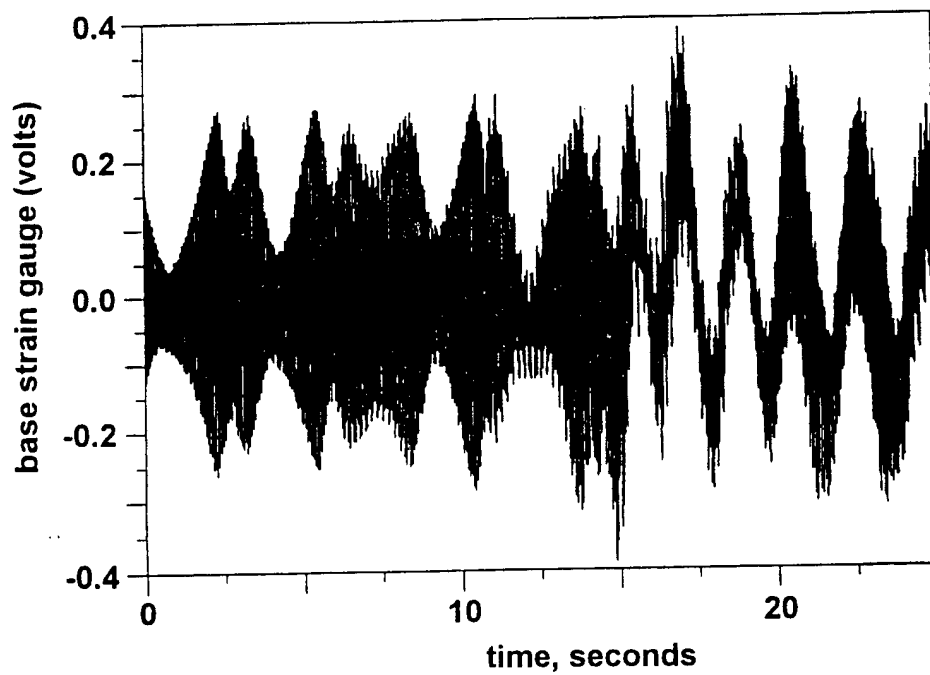

Figure 11: Strain-gage reading as a function of time for a parametrically excited cantilever beam, showing the transfer of energy from a high-frequency mode to a low-frequency mode (Anderson et al., 1992).

yet, the contribution of the first mode to the multifrequency response was much larger than the contribution of the high-frequency mode that was directly excited. In other words, the energy in the structure was transferred from the high-frequency mode to the low-frequency mode and, in that process, the response was transformed from a low-amplitude high-frequency motion into a high-amplitude low-frequency motion. Surprisingly small-amplitude excitations can produce this nonlinear phenomenon. Such changes are very undesirable because the increase in the amplitude of the motion can lead to a catastrophic failure. Exchanges of energy between widely spaced modes seem to be generic to a very broad class of dynamic systems, as seen in the next three sections.

2.3.2 Experiments on a Transversely Excited Cantilever Beam

In the second experiment, Tabaddor and Nayfeh (1995) transversely excited a steel cantilever beam with the dimensions 33.0" × 0.6" × 0.032" around its fourth natural frequency. The lowest five experimentally obtained natural frequencies are 0.70, 5.89, 16.75, 33.10, and 54.40 Hz.

The excitation amplitude was set at 1.0 g and the excitation frequency f was set at 30.2 Hz. The response consisted of a small-amplitude fourth mode. As f was slowly increased, the response continued to consist of only the fourth mode but with increasing amplitude. As f was increased past 32.40 Hz, a jump to a large-amplitude motion consisting of only the fourth mode occurred. As f was increased further, the response consisted of only the fourth mode but with decreasing amplitude.

Reversing the direction of sweep, Tabaddor and Nayfeh (1995) observed that the response continued to consist of only the fourth mode but with increasing amplitude until $f \approx 32.20$ Hz. As f was decreased below 32.20 Hz, they observed a response consisting of both the first and fourth modes. The time trace of the signal from a strain gage attached near the bottom of the beam is shown in Fig. 12a. Clearly the response consists of a high-frequency component riding over a low-frequency component. This is better seen in the spectrum of this signal shown in Fig. 12b. The spectrum consists of two major peaks corresponding to the fourth and first modes with asymmetric sidebands around the frequency of the fourth mode. The asymmetry of the sidebands indicates that the amplitude and phase of the fourth mode are modulated. Moreover, the spacing of the sidebands is equal to the frequency of the first mode. These results corroborate the results of the parametrically excited cantilever beam in that energy is transferred from the high-frequency fourth mode to the low-frequency first mode and that this transfer is accompanied by a modulation of the amplitude and phase of the high-frequency mode.

2.3.3 Frames

In the third experiment, Popovic, Nayfeh, Oh, and Nayfeh (1994) tested the frame shown in Fig. 5. The excitation was fixed at 1.2 g and at 195.6 Hz; the latter is close to the natural frequency of the eighth mode. The power spectrum of the response is shown in Fig. 13. One significant feature of the response power spectrum is the presence of sidebands around 195.6 Hz; the spacing of these sidebands is 3.7 Hz. The nonuniform distribution of these sidebands indicates that both the amplitude and the phase of the eighth mode were modulated. Observing the time trace of the signal, Fig. 14, one can see small-amplitude modulations of the eighth-mode response that never settled down. The power spectrum of the response in Fig. 12b indicates that the first and eighth modes were activated and that the amplitude and phase of the eighth mode were modulated.
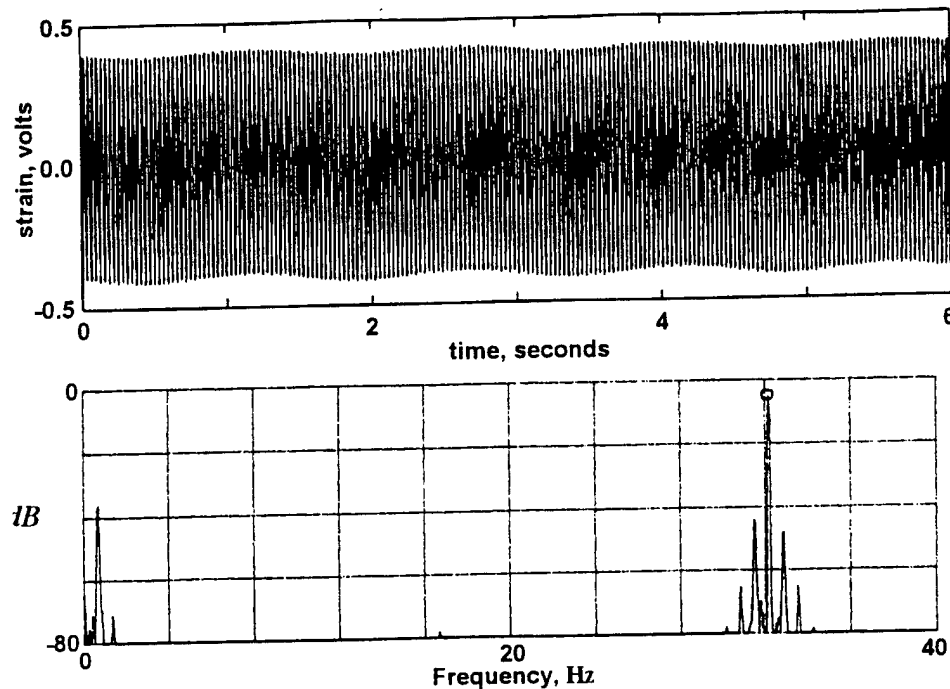
Figure 12: Energy transfer from the fourth mode to the first mode in a transversely excited cantilever beam: (a) time trace and (b) power spectrum (Tabaddor and Nayfeh, 1995).

2.3.4 Composite Plates

In the fourth experiment. Oh and Nayfeh (1995a) constructed a 12-layer graphite-epoxy plate 0.138" thick and 6.87" wide with the stacking sequence $(90/30/-30/-30/30/90)_s$ and tested it in a cantilever mode with a length of 11.50". The natural frequencies and mode shapes were determined by finite-element analysis and experimental modal analysis. The lowest seven experimentally identified natural frequencies are 42.79, 145.75, 275.11, 489.68, 734.66, 798.03, and 1001.4 Hz. This plate has widely spaced frequencies and some of the higher frequencies are commensurate. By changing the length of the plate, they were able to tune or detune some of the internal resonances and to determine their influence on the transfer of energy from high-frequency modes to the first mode.

The excitation was fixed at 12.92 g and 1001.4 Hz: the latter is equal to the natural frequency of the seventh mode. The power spectrum. Fig. 15b. contains two major peaks with frequencies close to those of the directly excited seventh mode and the first mode. One significant feature of this spectrum is the presence of a sideband around the major peak. corresponding to the seventh mode. The spacing of this sideband is 42.8 Hz. which is close to the frequency of the first mode. Again, we conclude that the low-frequency first mode is activated and that this activation is accompanied by a modulation of the amplitude and phase of the seventh mode. This is clear in the time trace in Fig. 15a.

2.3.5 A Paradigm for the Transfer of Energy Among Widely Space Modes

The results presented in Sections 4.1–4.4 show modal interactions occurring between high- and low-frequency modes in flexible as well as stiff structures. The mechanism for the interaction appears to be neither a classical internal resonance nor an external or parametric resonance involving the low-frequency modes. Rather, it seems that these interactions can occur whenever there exist modes
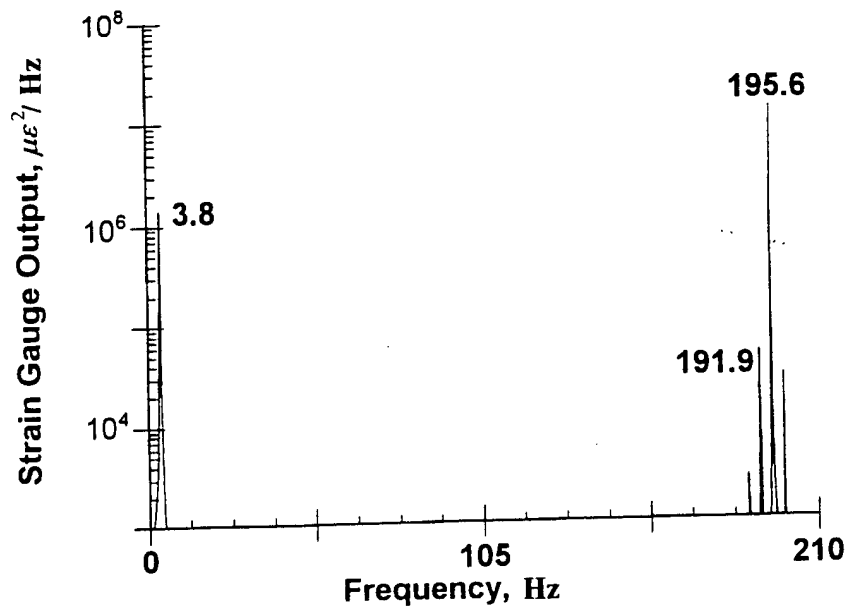
16

Figure 13: Spectrum of the response of the frame. showing the transfer of energy from the eighth mode to the first mode (Popovic et al.. 1994).
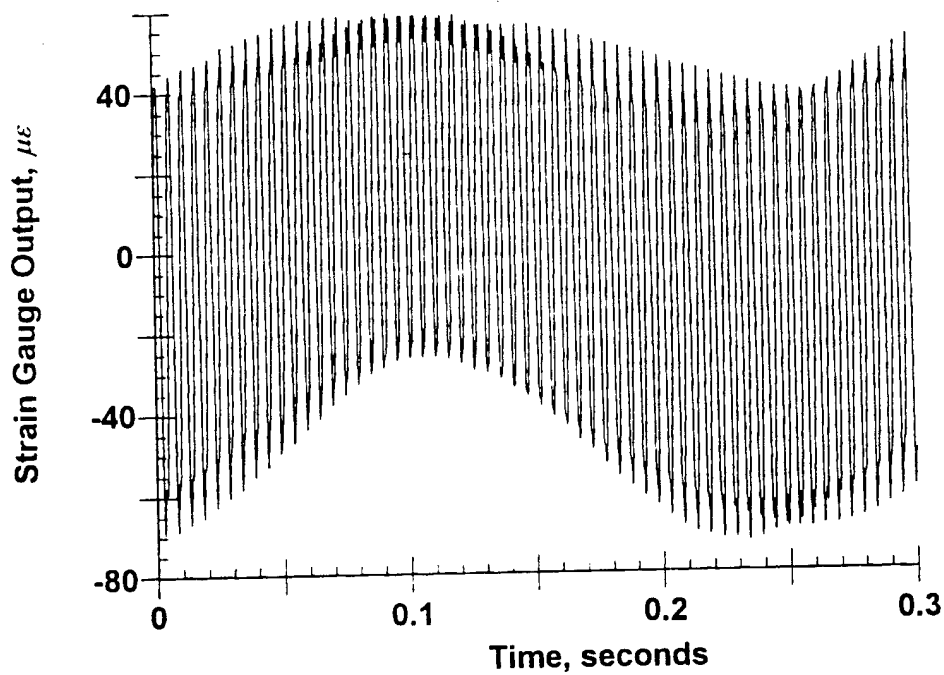

Figure 14: Time trace of the response of the frame. showing the transfer of energy from the eighth mode to the first mode (Popovic. et al.. 1994).
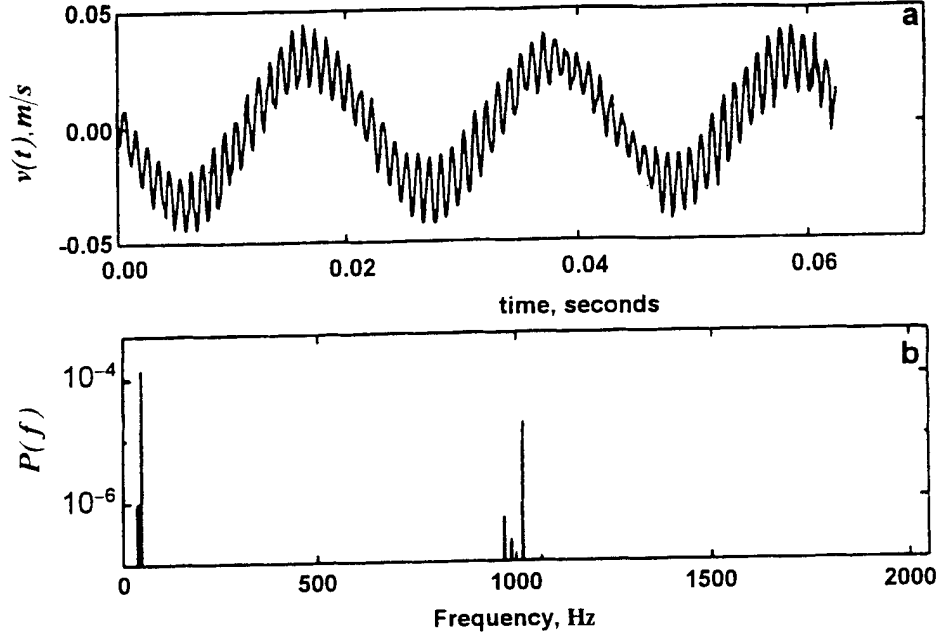
Figure 15: Response of a cantilever composite plate. showing the transfer of energy from the seventh mode to the first mode (Oh and Nayfeh, 1995a).

whose natural frequencies are much lower than the natural frequencies of the modes being directly driven.

To investigate possible mechanisms for the transfer of energy from high- to low-frequency modes. Nayfeh and Nayfeh (1993) studied a representative system made up of two coupled oscillators. These equations are of a form that may be obtained by a two-mode discretization of a continuous system with cubic nonlinearities or. alternatively. they could model a discrete two-degree-of-freedom system. The equations are given by

$$\ddot{u}_1 + 2\epsilon\mu_1\dot{u}_1 + \epsilon^2 u_1 = -\epsilon^2(4\alpha_1 u_1^3 + \alpha_2 u_1 u_2^2) \tag{1}$$

$$\ddot{u}_2 + 2\epsilon\mu_2\dot{u}_2 + u_2 = \epsilon(\alpha_3 u_2^3 + \alpha_4 u_1^2 u_2 + f\cos\Omega t) \tag{2}$$

where ϵ. the ratio of the linear natural frequencies of the system. is positive and small. The linear natural frequency of the high-frequency mode has been nondimensionalized to unity. Of principal interest is whether an excitation applied to the high-frequency mode near its linear natural frequency can, as observed in the experiments, generate a large response in the low-frequency mode. To answer this question. Nayfeh and Nayfeh (1993) used the method of averaging to construct an approximation to the solutions of Eqs. (1) and (2).

Neglecting the damping and nonlinearities. one can write the solution to Eq. (1) as $u_1 = A_0\cos(\epsilon t + \phi_0)$. In this solution. u_1 is $O(1)$, $\dot{u}_1$ is $O(\epsilon)$, and $\ddot{u}_1$ is $O(\epsilon^2)$. Thus, u_1 itself is slowly varying. Because the natural frequency of u_2 is not small. its motion can be treated in the usual way by assuming that its amplitude and phase are slowly varying as described below.

It is convenient to set $\Omega^2 = 1 + \epsilon\sigma$. where σ is a measure of the closeness of the excitation frequency to the unperturbed natural frequency of u_2. It follows from the method of averaging that

$$u_2 = a(t)\cos(\Omega t + \beta(t)) \quad \text{and} \quad \dot{u}_2 = -a(t)\Omega\sin(\Omega t + \beta(t)) \tag{3}$$

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where

$$\dot{a} = -\epsilon(\mu_2 a + \frac{1}{2}f \sin \beta) \tag{4}$$

$$\dot{\beta} = -\epsilon \left(\frac{1}{2}\sigma + \frac{1}{2}\alpha_4 u_1^2 + \frac{3}{8}\alpha_3 a^2 + \frac{f}{2a} \cos \beta \right) \tag{5}$$

$$\dot{u}_1 = \epsilon v_1 \tag{6}$$

$$\dot{v}_1 = -\epsilon(u_1 + 2\mu_1 v_1 + 4\alpha_1 u_1^3 + \frac{1}{2}\alpha_2 u_1 a^2) \tag{7}$$

The fixed-point solutions of the averaged equations represent constant amplitude and phase motions of the high-frequency mode accompanied by static (DC) responses of the low-frequency mode. The stability of a fixed-point solution is determined by examination of the eigenvalues of the Jacobian matrix of Eqs. (4)-(7) evaluated at the fixed point of interest.

In Fig. 16, the frequency-response curves are presented for a case in which nontrivial solutions for u_1 occur. It should be noted that, although we show only the fixed-point solutions corresponding to positive values of u_1 in Fig. 16, there exists a second set of solutions corresponding to negative values of u_1. The trivial solutions are unstable with a positive real eigenvalue in the central region of the plot. In this region, a nontrivial solution for u_1 exists. The upper branch of this solution consists of two regions of stable nodes joined by a region of unstable foci. Where the stable nodes exist, the motion will consist of periodic oscillations in u_2 and either a positive or negative nonzero static deflection in u_1. Where the unstable foci exist, oscillatory u_1 motions accompanied by modulated u_2 responses will occur as observed in the experiments.

The averaged equations were integrated for the same values of the parameters used in Fig. 16 to study the dynamics of the system in the neighborhoods of unstable foci. As predicted by the stability analysis, oscillatory responses of u_1 were found. The dynamics of the system are very complicated in these regimes and various nonlinear phenomena, such as period-doubling bifurcations culminating in chaos, symmetry-breaking bifurcations, the existence of multiple attractors, and the merging of attractors were found.

In Fig. 17, a sequence of responses obtained for the parameter values used in Fig. 16, $f = 2.5$, and various values of σ is presented. As shown in Fig. 16, as σ is decreased through $\sigma = 0.349$, a Hopf bifurcation occurs. In Fig. 16a, we plot the motion in the $a - u_1$ plane just before the supercritical Hopf bifurcation occurs. As expected, the long-time response consists of only the stable fixed point. There exists a second fixed-point solution corresponding to negative values of u_1 which is not plotted here.

In Fig. 17b, the motion just after the bifurcation is shown. As predicted, the response changes from the point in the plane shown in Fig. 17a to the limit cycle in Fig. 17b. As σ is further decreased, the size of the limit cycle increases, Figs. 17b-e. Decreasing σ further leads to the sequence of period-doubling bifurcations shown in Figs. 17f and g, which culminates in the creation of the chaotic attractor shown in Fig. 17h. Only a short sample of the chaotic attractor is shown. As the motion continues, the trajectory would fill the area outlined roughly by the portion of the trajectory shown.

The analysis of this two-degree-of-freedom system shows interactions between high- and low-frequency modes through which an excitation applied to the high-frequency mode results in large-amplitude responses in the low-frequency mode. The response of the system is similar to that reported in the two experiments.

Nayfeh and Chin (1995) extended the analysis of Nayfeh and Nayfeh (1993) by considering the case of principal parametric resonance. They showed that exciting the high-frequency mode with a principal parametric resonance results in a large-amplitude response in the low-frequency mode, again similar to the experimental observations.
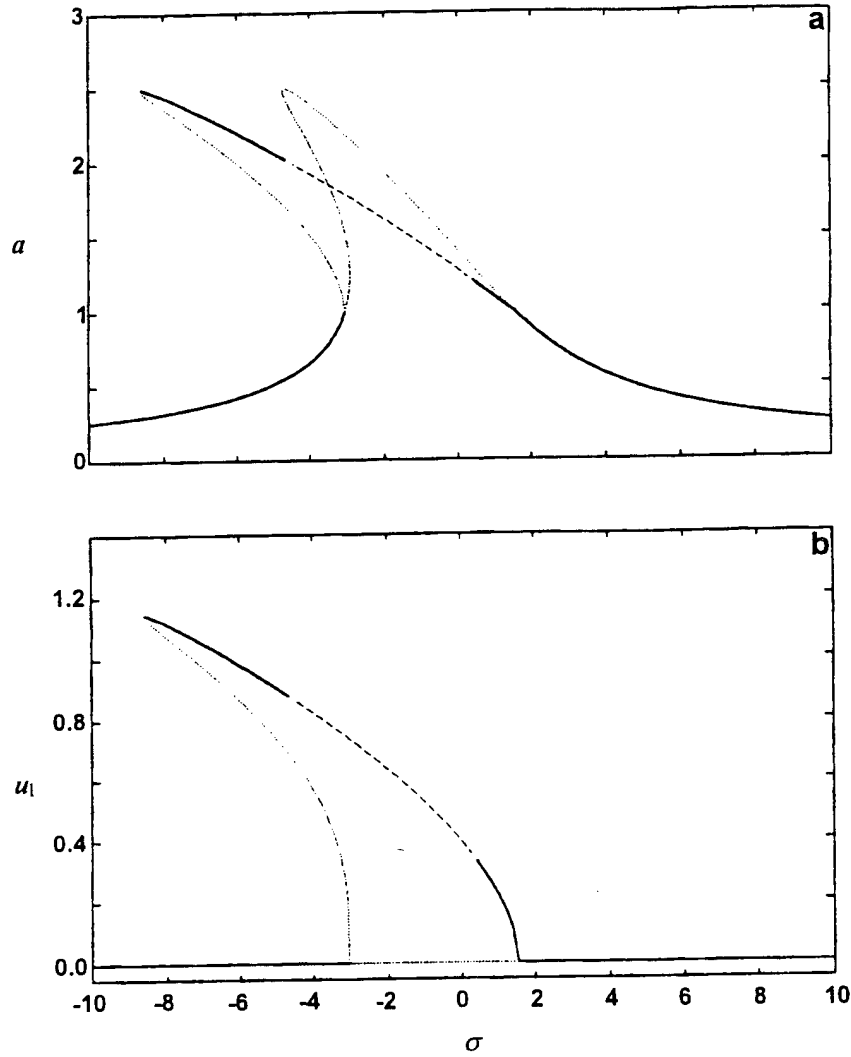
Figure 16: Frequency-response curves for $\alpha_1 = \alpha_3 = 1. \alpha_2 = -2. \alpha_4 = 3. \mu_1 = 0.25. \mu_2 = 0.5$, and $f = 2.5$. Solid lines denote stable solutions. dotted lines denote unstable solutions with a positive real eigenvalue, and dashed lines denote unstable solutions with a complex-conjugate pair of eigenvalues in the right half-plane (Nayfeh and Nayfeh. 1993).
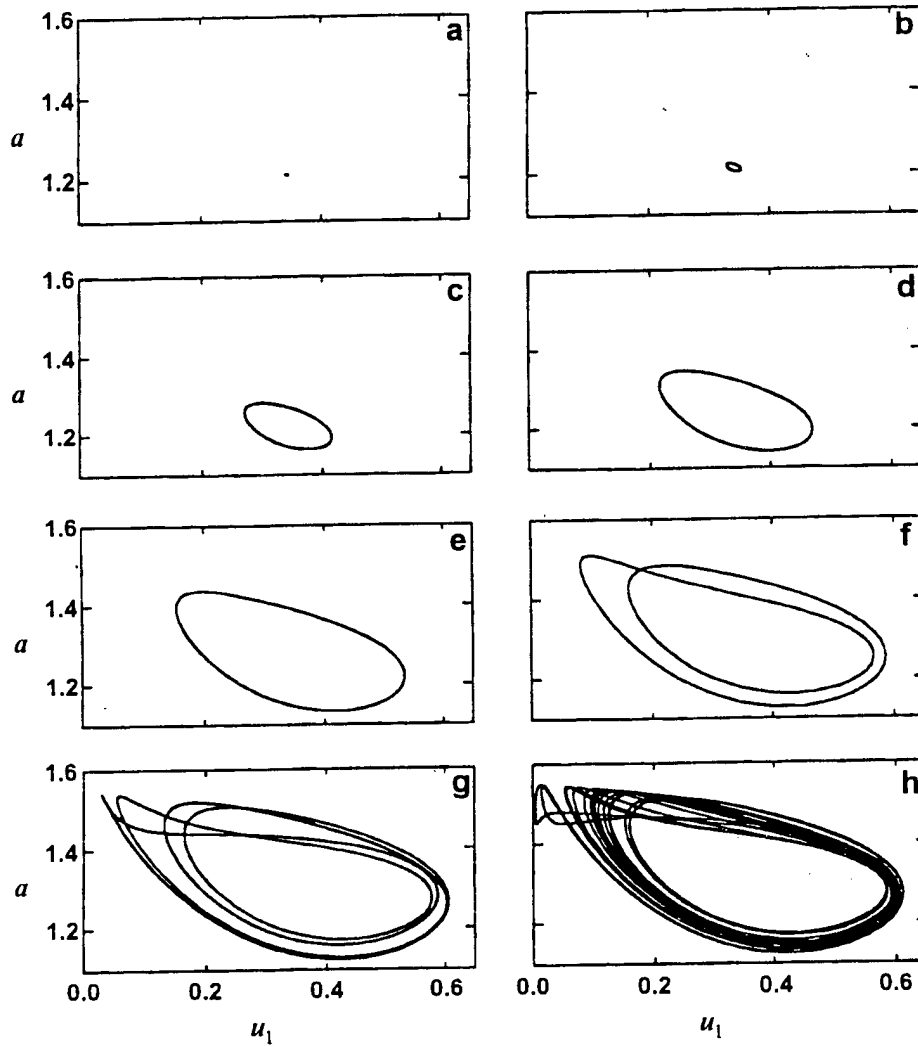
Figure 17: Numerical simulation of the averaged equations for $\alpha_1 = \alpha_3 = 1. \alpha_2 = -2. \alpha_4 = 3.$ $\mu_1 = 0.25, \mu_2 = 0.5, f = 2.5,$ and $\sigma =$ (a) 0.350. (b) 0.348. (c) 0.300. (d) 0.200. (e) 0.000. (f) -0.170, (g) -0.243, and (h) -0.260 (Nayfeh and Nayfeh. 1993).

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3   Students Receiving Full or Partial Support

The following students received full or partial support from the AFOSR Grants. The support took the form of stipends paid directly to them and/or money paid to the PIs to release them from other academic duties.

Senior Projects

P. Popovic, 1994, "An Experimental Investigation of Energy Transfer from a High-Frequency Mode to a Low-Frequency Mode in a Flexible Structure"

Masters Students

1. S. A. Nayfeh, 1993. "Nonlinear Dynamics of Systems Involving Widely Spaced Frequencies

2. W. Kreider. 1995, "Linear and Nonlinear Vibrations of Buckled Beams"

Doctoral Students

1. T. J. Anderson, 1993, "Nonlinear Vibrations of Metallic and Composite Structures"

2. M. U. Bikdash. 1993, "Time-Optimal and Saturating Controls with Applications to Space Structure"

3. C-M. Chin. 1993, "Problems in Nonlinear Dynamics"

4. K. Oh. 1994. "A Theoretical and Experimental Study of Modal Interactions in Metallic and Laminated Composite Plates"

4   Publications

The following articles were made possible by full or partial support from the AFOSR Grants.

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1. A. M. Abou-Rayan, A. H. Nayfeh, D. T. Mook. and M. A. Nayfeh. "Nonlinear Response of a Parametrically Excited Buckled Beam." Nonlinear Dynamics. Vol. 4. No. 5. 1993, pp. 499-525.

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